

Zestaw nr 11. Przekształcenia liniowe

1. Wskazać bazy i określić wymiary podanych przestrzeni wektorowych:

- (a) $\{[2x, x + y, 3x - y, x - 2y] \mid x, y \in \mathbb{R}\} \subseteq \mathbb{R}^4$
- (b) $\{[x - 2y - z, 2x + y - 3z, 3x + 4y - 5z] \mid x, y, z \in \mathbb{R}\} \subseteq \mathbb{R}^3$
- (c) $\{[x, y, z, t] \mid x + y = z - y\} \subseteq \mathbb{R}^4$
- (d) $\{[x + y + z, x - y, x - z, y - z] \mid x, y, z \in \mathbb{R}\} \subseteq \mathbb{R}^4$

2. Które z następujących przekształceń są liniowe?

- (a) $F: \mathbb{R}^3 \rightarrow \mathbb{R}^2, F([x_1, x_2, x_3]) = [x_1, x_3]$
- (b) $F: \mathbb{R}^3 \rightarrow \mathbb{R}^2, F([x_1, x_2, x_3]) = [x_1 + x_2, 2x_1 - x_2 + 3x_3]$
- (c) $F: \mathbb{R} \rightarrow \mathbb{R}, F(x) = (x + 1)(x - 1)$
- (d) $F: \mathbb{R}^2 \rightarrow \mathbb{R}^2, F([x_1, x_2]) = [3x_1 + x_2 - 1, 2x_1 - 3x_2]$
- (e) $F: \mathbb{R}^2 \rightarrow \mathbb{R}^3, F([x_1, x_2]) = [x_1 + 2x_2, x_1 - x_2, x_1]$
- (f) $F: \mathbb{R}^2 \rightarrow \mathbb{R}, F([x_1, x_2]) = x_1 \cdot x_2$
- (g) $F: \mathbb{R}^3 \rightarrow \mathbb{R}^3, F([x_1, x_2, x_3]) = [x_1 + 1, x_2 + 2, x_3 + 3]$
- (h) $F: \mathbb{R} \rightarrow \mathbb{R}, F(x) = |x|$
- (i) $F: \mathbb{R}^3 \rightarrow \mathbb{R}^3, F([x_1, x_2, x_3]) = [x_1 x_2, x_1, x_3]$
- (j) $F: \mathbb{R}^2 \rightarrow \mathbb{R}^3, F([x_1, x_2]) = [2x_1 - x_2, x_1 + 1, x_2 - 1]$
- (k) $F: \mathbb{R}^2 \rightarrow \mathbb{R}^2, F([x_1, x_2]) = [2x_1 + x_2, x_2]$
- (l) $F: \mathbb{R}^2 \rightarrow \mathbb{R}^2, F([x_1, x_2]) = [x_1^2, x_2]$
- (m) $F: \mathbb{R}^4 \rightarrow \mathbb{R}^4, F([x_1, x_2, x_3, x_4]) = [-x_1, -x_2, -x_3, -x_4]$

3. Dla każdego z przekształceń wyznaczyć jądro $\text{Ker} F$ i obraz $\text{Im} F$. Podać bazy i wymiary tych podprzestrzeni.

- (a) $F: \mathbb{R}^3 \rightarrow \mathbb{R}^2, F([x_1, x_2, x_3]) = [x_1 + x_2, x_2 + x_3]$
- (b) $F: \mathbb{R}^3 \rightarrow \mathbb{R}^4, F([x_1, x_2, x_3]) = [2x_1 - x_2 + x_3, x_1 + 2x_2 - x_3, -x_1 + 3x_2 - 2x_3, 8x_1 + x_2 + x_3]$
- (c) $F: \mathbb{R}^2 \rightarrow \mathbb{R}^2, F([x_1, x_2]) = [2x_1 - x_2, 3x_2 - 6x_1]$
- (d) $F: \mathbb{R}^3 \rightarrow \mathbb{R}^4, F([x_1, x_2, x_3]) = [2x_1 - x_2 - x_3, x_1 + x_2 + 4x_3, 2x_1 + x_2 + 5x_3, -x_1 - x_3]$
- (e) $F: \mathbb{R}^4 \rightarrow \mathbb{R}^3, F([x_1, x_2, x_3, x_4]) = [x_1 + 2x_3 + x_4, -2x_1 + x_2 - 3x_3 - 5x_4, x_1 - x_2 + x_3 + 4x_4]$
- (f) $F: \mathbb{R}^4 \rightarrow \mathbb{R}^5, F([x_1, x_2, x_3, x_4]) = [x_1 + x_2, x_2 + x_3, x_3 + x_4, x_3, x_1]$
- (g) $F: \mathbb{R}^4 \rightarrow \mathbb{R}^3, F([x_1, x_2, x_3, x_4]) = [2x_1 + x_3, 2x_2 - x_4, x_3 + 2x_4]$
- (h) $F: \mathbb{R}^4 \rightarrow \mathbb{R}^4, F([x_1, x_2, x_3, x_4]) = [x_4, x_3, x_2, x_1]$
- (i) $F: \mathbb{R}^4 \rightarrow \mathbb{R}^3, F([x_1, x_2, x_3, x_4]) = [x_1 + x_2 + x_3 - x_4, 2x_1 + x_2 - x_3 + x_4, x_2 + 3x_3 - 3x_4]$
- (j) $F: \mathbb{R}^5 \rightarrow \mathbb{R}^3, F([x_1, x_2, x_3, x_4, x_5]) = [x_1 + x_2 + x_3, x_2 + x_3 + x_4, x_3 + x_4 + x_5]$
- (k) $F: \mathbb{R}^4 \rightarrow \mathbb{R}^3, F([x_1, x_2, x_3, x_4]) = [x_1 + x_3, x_2 - x_4, 2x_3]$
- (l) $F: \mathbb{R}^4 \rightarrow \mathbb{R}^4, F([x_1, x_2, x_3, x_4]) = [x_1 - 2x_2 + 3x_3 - 4x_4, 3x_1 + 5x_3 + 2x_4, x_1 + x_2 + x_3 + 3x_4, 5x_1 - x_2 + 9x_3 + x_4]$
- (m) $F: \mathbb{R}^3 \rightarrow \mathbb{R}^2, F([x_1, x_2, x_3]) = [x_1 - 3x_2 + 2x_3, -2x_1 + 6x_2 - 4x_3]$
- (n) $F: \mathbb{R}^4 \rightarrow \mathbb{R}^3, F([x_1, x_2, x_3, x_4]) = [x_1 + 2x_2 + x_3 - x_4, x_1 + 2x_3 + x_4, 2x_1 + x_2 + 3x_3]$
- (o) $F: \mathbb{R}^4 \rightarrow \mathbb{R}^3, F([x_1, x_2, x_3, x_4]) = [x_1 - x_3, 3x_2, x_4 - 2x_2]$
- (p) $F: \mathbb{R}^4 \rightarrow \mathbb{R}^3, F([x_1, x_2, x_3, x_4]) = [x_1 + x_3, x_2 - x_4, x_1 + x_2 + x_3 + x_4]$
- (q) $F: \mathbb{R}^5 \rightarrow \mathbb{R}^3, F([x_1, x_2, x_3, x_4, x_5]) = [x_1 - x_2 + x_3 - x_4 - x_5, -x_1 + x_2 + 2x_4 + 4x_5, -2x_1 + 2x_2 - 2x_3 + 3x_4 + x_5]$